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The Donnachie-Landshoff pomeron and gauge invariance

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Abstract. The pomeron model of Donnachie and Landshoff does not conserve the electromagnetic current when applied to diffractive reactions such as electroproduction of a quark-antiquark pair or of a vector meson. We propose a treatment of this problem which ensures a physical behaviour of cross sections in the photoproduction limit and show that it leads to results rather similar to those obtained from two-gluon exchange.

1 Introduction

The description of the pomeron within the framework of QCD remains one of the great tasks in strong interaction physics. Important progress has been made in this field over the last years, especially in the study of diffractive processes with a hard scale, for instance a large photon virtuality Q^2 in ep collisions. On the other hand, phenomenological models are still of interest in this new dynamical regime: calculations in the framework of multi-gluon exchange can be of considerable complexity, and it is useful to have models that reproduce their results in a simpler way, thus allowing the investigation of more complicated reactions with a reasonable amount of effort. Furthermore, such models provide an opportunity to extrapolate from the hard to the soft diffractive regime, e.g. to take the limit of small Q^2 in diffractive ep scattering, and to make contact with elastic and diffractive hadron scattering, domains where the application of QCD is much more difficult since perturbation theory is of little help. It goes by itself that one cannot expect such models to reproduce *all* results and aspects of more sophisticated and more fundamental QCD approaches.

In this paper we are concerned with the pomeron model of Donnachie and Landshoff (DL) [1] and its confrontation with the QCD motivated model of nonperturbative gluon exchange by Landshoff and Nachtmann (LN) [2, 3]. It was found in [4] that the DL model has problems with electromagnetic gauge invariance, which appear for instance when it is applied to the production of a quark-antiquark pair in diffractive ep scattering. In Sect. 2 we explain the origin of this problem and propose a possible solution which makes the DL model reproduce several, although not all results of the two-gluon calculation in the LN approach. We then investigate exclusive vector meson production under the same aspects. In Sect. 4 we discuss our way to handle the gauge invariance problem from the point of view of contact terms, and make some comparison with other approaches in the literature. We summarise our results in Sect. 5.

2 Diffractive $q\bar{q}$-production

2.1 The problem in the DL model and a solution

In the DL model the pomeron couples to quarks via their vector current, a choice originally motivated by the analysis of elastic and diffractive hadron-hadron scattering [5]. In this sense the pomeron is said to behave like a photon, *except* that it has charge conjugation parity $C = +1$ unlike the photon with $C = -1$. To implement this difference of quantum numbers the model prescribes to subtract Feynman diagrams which are related by reversing the charge flow of quark lines, instead of adding them as one would do if the pomeron were replaced by a photon.

Calculating the forward $\gamma^* p$ scattering amplitude at high energy in this model DL found that in order to obtain Bjorken scaling of the proton structure function at small x the quark-pomeron vertex cannot be pointlike but must be softened at large quark virtualities [1]. They made the ansatz

$$\beta_0 \gamma^\mu f(k_1^2 - m_q^2)\, f(k_2^2 - m_q^2) \qquad (1)$$

with

$$f(k^2) = \mu_0^2 / (\mu_0^2 - k^2) \qquad (2)$$

for the pomeron coupling to quarks of mass m_q and momenta k_1 and k_2, with the constants $\beta_0 \approx 2\ \mathrm{GeV}^{-1}$ and $\mu_0 \approx 1$ GeV determined from phenomenology.

We now investigate diffractive $q\bar{q}$-production,

$$\gamma^*(q) + p(p) \to q(P_q) + \bar{q}(P_{\bar{q}}) + p(p') \ , \qquad (3)$$

with four-momenta given in parentheses. We will further use the variables $\Delta = p - p'$, $t = \Delta^2$, $W^2 = (p + q)^2$, $Q^2 = -q^2$, $M^2 = (P_q + P_{\bar{q}})^2$, $\beta = Q^2/(2\Delta \cdot q)$ and $\xi = (\Delta \cdot q)/(p \cdot q)$. In the DL model this reaction is described by the two diagrams of Fig. 1, taken with opposite relative sign. One finds [4] that the electromagnetic current is not conserved here: the $\gamma^* p$ cross section σ_U for the unphysical photon polarisation $\varepsilon_3^\mu = q^\mu / Q$ does not vanish but instead has a $1/Q^2$ singularity in the photoproduction limit.

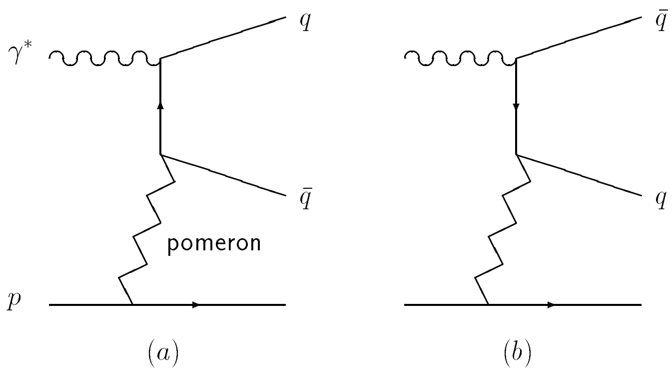

Fig. 1. Diffractive $q\bar{q}$-production in the DL model. The relative sign of the diagrams is the opposite of what it would be if the pomeron were replaced by a photon, because the pomeron and photon have opposite charge conjugation parity

In a technical sense the reason is that the two diagrams in Fig. 1 have the "wrong" relative sign which was assigned in order to implement the correct charge conjugation parity of the pomeron – for photon instead of pomeron exchange gauge invariance is of course guaranteed. If one uses the Feynman gauge for the photon field and defines longitudinal and transverse photon polarisations with respect to the γ^*p axis in the c.m. of the collision then the cross section σ_L for longitudinal photons has the same unphysical $1/Q^2$ behaviour at small Q^2, only the transverse cross section σ_T is not affected by this problem.

It is instructive to look at the dependence of the cross sections on the transverse momentum p_t of the produced quark in the γ^*p c.m. and on the invariant mass M of the $q\bar{q}$-pair. Taking quark masses equal to zero the unphysical and longitudinal cross sections have a factor p_t^2/M^2 relative to the transverse one, and as a consequence one finds that at large Q^2 the p_t-integrated cross sections $d\sigma_U/(dM^2)$ and $d\sigma_L/(dM^2)$ are suppressed by $\mu_0^2/Q^2 \cdot \log(Q^2/\mu_0^2)$ compared with $d\sigma_T/(dM^2)$. In this sense one can say that the gauge violating terms are of higher twist and that the model can be used for leading twist quantities. In jet production, where p_t^2/M^2 is not small, the unphysical cross section is however not negligible compared with $d\sigma_T/(dp_t^2\, dM^2)$, even at large Q^2, and the model is of no use as it stands.

Let us have a closer look at the origin of the bad Q^2-behaviour of $d\sigma_L/(dp_t^2\, dM^2)$. From the expression of the longitudinal polarisation vector in Feynman gauge

$$\varepsilon_0 = \frac{1}{\sqrt{1 + p^2\, Q^2/(p\cdot q)^2}} \left(\frac{1}{Q}\, q + \frac{Q}{p\cdot q}\, p \right) \tag{4}$$

it is clear that the part of ε_0 proportional to p gives a factor Q in the amplitude and therefore a factor Q^2 in the γ^*p cross section as required by gauge invariance. The unphysical behaviour comes from the part proportional to q, which would give no contribution if the electromagnetic current were conserved in the model.

Without current conservation the results of a calculation will clearly be gauge dependent. Therefore we need to *fix* a photon gauge in order to make the model well defined. We feel that this is legitimate in the context of a phenomenological model, a choice of gauge then has to be justified by its results. We choose to work in a noncovariant gauge with gauge fixing vector p, where the photon field satisfies $A \cdot p = 0$. In this gauge a polarisation vector ε in Feynman gauge becomes

$$\varepsilon \to \tilde{\varepsilon} = \varepsilon - \frac{p\cdot\varepsilon}{p\cdot q}\, q \ . \tag{5}$$

It is instructive to compare the tensor structure of the photon propagator,

$$g^{\mu\nu} = -\mathrm{sgn}(q^2)\, \varepsilon_0^\mu \varepsilon_0^\nu - \varepsilon_1^\mu \varepsilon_1^\nu - \varepsilon_2^\mu \varepsilon_2^\nu + \mathrm{sgn}(q^2)\, \varepsilon_3^\mu \varepsilon_3^\nu \tag{6}$$

in Feynman gauge and

$$\begin{aligned} g^{\mu\nu} - \frac{1}{p\cdot q}\, (p^\mu q^\nu + q^\mu p^\nu) + \frac{p^2}{(p\cdot q)^2}\, q^\mu q^\nu \\ = -\mathrm{sgn}(q^2)\, \tilde{\varepsilon}_0^\mu \tilde{\varepsilon}_0^\nu - \tilde{\varepsilon}_1^\mu \tilde{\varepsilon}_1^\nu - \tilde{\varepsilon}_2^\mu \tilde{\varepsilon}_2^\nu \end{aligned} \tag{7}$$

in our noncovariant gauge, where ε_1 and ε_2 are two orthogonal vectors transverse to p and q. We see that in the amplitude for the electroproduction process $ep \to ep + q\bar{q}$ the term $\varepsilon_3^\mu \varepsilon_3^\nu$ on the r.h.s. of (6) drops out when the index μ is contracted with the electron current since this current is conserved, so that we are left with the ill behaved contribution from the contraction of q^ν/Q in ε_0^ν with the current of the produced quarks. In the gauge (5) the transverse polarisations, which dominate in the p_t-integrated cross sections at large Q^2, are the same as before, $\tilde{\varepsilon}_1 = \varepsilon_1$ and $\tilde{\varepsilon}_2 = \varepsilon_2$, whereas the longitudinal polarisation

$$\tilde{\varepsilon}_0 = \frac{1}{\sqrt{1 + p^2\, Q^2/(p\cdot q)^2}} \cdot \frac{Q}{p\cdot q} \left(p - \frac{p^2}{p\cdot q}\, q \right) \tag{8}$$

behaves like Q in the photoproduction limit and leads to a reasonable behaviour of the amplitude. From the l.h.s. of (7) we see that when changing from Feynman to our noncovariant gauge we have effectively added the contraction of $-p^\mu q^\nu/(p \cdot q)$ with the leptonic and hadronic currents to the $ep \to ep + q\bar{q}$ amplitude, the terms with q^μ in the photon propagator giving zero when contracted with the electron current.

Instead of (5) one may consider other noncovariant gauges $A \cdot n = 0$ with a suitably chosen vector n. Introducing longitudinal and transverse photon polarisations with respect to n and q and repeating the arguments that lead from (4) to (8) one will find that the ep amplitude has a well behaved photoproduction limit. This amplitude will in general be different for different choices of n since one has effectively added $-n^\mu q^\nu/(n \cdot q)$ to the tensor $g^{\mu\nu}$ in the photon propagator. Note that the γ^*p amplitude for photons that are transverse with respect to p and q will in general also be modified by such a choice of gauge.

A candidate gauge fixing vector is $n = \Delta$, which for $t = 0$ is equivalent to $n = p$ since then $\Delta = \xi p$, but leads to different results at finite t. Taking $n = \Delta$ looks more symmetric with respect to the photon dissociation and proton scattering parts of our reaction; to leading order in ξ^{-1} the choice $n = p$ is however equivalent to $n = p + q$, which is also symmetric. We shall come back to the question of p versus Δ in Sect. 3.

2.2 Comparison of the DL and LN models

We shall now see that in the gauge (5) the predictions of the DL model for our process are remarkably similar to those in the LN model, although they are not identical. We restrict ourselves to $t = 0$ but admit a finite quark mass m_q in the produced $q\bar{q}$-pair. With Hand's convention for the flux factor the cross section of the process (3) for a given photon polarisation reads

$$\left.\frac{d\sigma}{dt\, dp_t^2\, dM^2}\right|_{t=0} = \frac{3}{128\pi^3} \frac{\xi^2}{(Q^2+M^2)^2\, M^2 \sqrt{1-4(p_t^2+m_q^2)/M^2}} \times {\sum_{spins}}' |\mathcal{M}(\tilde{\varepsilon})|^2 \quad , \tag{9}$$

where ${\sum}'_{spins}$ stands for spin summation for the final state particles and spin average for the initial proton. In the DL model one has

$$\mathcal{M}(\tilde{\varepsilon}) = \frac{16\pi e e_q}{3} \frac{j\cdot q}{2p\cdot q} \xi^{-\alpha_{I\!P}(0)} \cdot K\, \bar{u}(P_q)\, \gamma\cdot\tilde{\varepsilon}\, v(P_{\bar{q}}) \tag{10}$$

with

$$K = \frac{9\beta_0^2}{4\pi} \frac{f(\hat{t}-m_q^2)+f(\hat{u}-m_q^2)}{2} \quad . \tag{11}$$

Here $\hat{t} = (q-P_q)^2$ and $\hat{u} = (q-P_{\bar{q}})^2$ are the Mandelstam variables of the pomeron-photon subreaction, e_q is the charge of the produced quark in units of the positron charge e and $\alpha_{I\!P}(t)$ is the soft pomeron trajectory.

$$j^\mu = \bar{u}(p') \left[F_1(t)\gamma^\mu - \frac{i}{2m_p}\sigma^{\mu\nu}\Delta_\nu F_2(t) \right] u(p) \tag{12}$$

is the isoscalar nucleon vector current with F_1 and F_2 being the isoscalar Dirac and Pauli nucleon form factors, i.e. the sum of the respective form factors of proton and neutron.

In the LN model the reaction (3) is described by the diagrams of Fig. 2 plus those obtained by reversing the charge flow of the upper quark line; for details of the calculation cf. [4]. Key quantities of the model are the moments

$$\int_0^\infty dl^2 [\alpha_s^{(0)} D(-l^2)]^2 = \frac{9\beta_0^2}{4\pi}$$
$$\int_0^\infty dl^2 [\alpha_s^{(0)} D(-l^2)]^2 \cdot l^2 = \frac{9\beta_0^2\mu_0^2}{8\pi} \tag{13}$$

of the nonperturbative gluon propagator $D(l^2)$, where the strong coupling in the nonperturbative region is taken as $\alpha_s^{(0)} \approx 1$ following [6]. β_0 and μ_0 have been identified in [3] with the corresponding parameters in the DL model, cf. (1) and (2). Note that μ_0^2 has here the significance of a typical gluon virtuality l^2 dominating in the integrals.

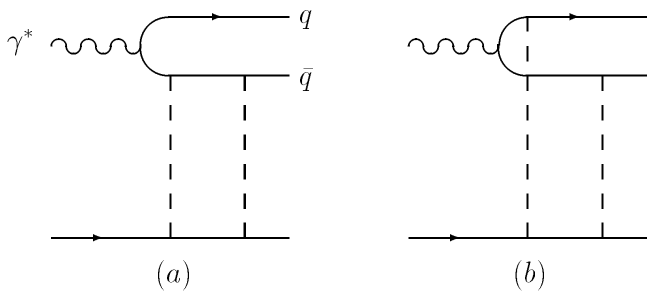

Fig. 2. Two of the four diagrams for $q\bar{q}$-production in the LN model, the other two are obtained by reversing the charge flow of the upper quark line. The lower line stands for a constituent quark in the proton and the pomeron is approximated by two nonperturbative gluons, represented by dashed lines

For diffractive $q\bar{q}$-production we have in our noncovariant gauge

$$\mathcal{M}(\tilde{\varepsilon}) = \frac{16\pi e e_q}{3} \frac{j\cdot q}{2p\cdot q} \xi^{-\alpha_{I\!P}(0)} \sqrt{\frac{\alpha_s(\lambda^2)}{\alpha_s^{(0)}}} \times \Bigg[L_1\, \bar{u}(P_q)\, \gamma\cdot\tilde{\varepsilon}\, v(P_{\bar{q}}) - (L_1-L_2) \frac{\tilde{\varepsilon}\cdot q}{\Delta\cdot q} \bar{u}(P_q)\, \gamma\cdot\Delta\, v(P_{\bar{q}}) + (L_1-L_2)\frac{1}{\lambda^2} m_q\, \bar{u}(P_q)\, \gamma\cdot\tilde{\varepsilon}\, \gamma\cdot\Delta\, v(P_{\bar{q}}) \Bigg] \quad , \tag{14}$$

where

$$L_i = \int_0^\infty dl^2 \left[\alpha_s^{(0)} D(-l^2)\right]^2 f_i(l^2, p_t^2, \lambda^2) \, , \quad i = 1,2 \tag{15}$$

are loop integrals with weighting functions f_1, f_2 whose expressions can be found in [4]. The variable

$$\lambda^2 = \frac{p_t^2+m_q^2}{1-\beta} \tag{16}$$

can be seen as the relevant hard scale of the process [7, 8]. The square root of $\alpha_s(\lambda^2)/\alpha_s^{(0)}$ in (14) corresponds to taking the quark-gluon coupling at a perturbative scale only for the upper left vertex in the diagrams of Fig. 2, where one quark leg has a virtuality of order λ^2. This choice was made in [9] and [4,10,11]. One may argue that the coupling should be taken as perturbative at both upper vertices, then there is no square root in (14), (17). A similar comment holds for vector meson production (30).

At this point we remark that many features of the LN model are due to the two-gluon exchange mechanism and in common with other two-gluon approaches. The authors of [8] and [12] use the gluon density in the proton to describe its coupling to the two exchanged gluons; at $t = 0$ their expressions are related to those in the LN model by the substitution[1]

$$\frac{\pi}{4}\alpha_s \cdot \frac{\partial}{\partial l^2} \left[\xi\, g(\xi, l^2)\right]$$

[1] This substitution is consistent with [6], where the gluon density was estimated within the LN model

$$\to \sqrt{\frac{\alpha_s(\lambda^2)}{\alpha_s^{(0)}}} \cdot \xi^{1-\alpha_{I\!P}(0)} \, [\alpha_s^{(0)} D(-l^2)]^2 \cdot l^2 \quad , \qquad (17)$$

where $g(\xi, l^2)$ is the gluon density at a factorisation scale l^2 and α_s on the l.h.s. taken at scale λ^2 in [8] and l^2 in [12]. The principal difference in the predictions of the two approaches is thus the ξ-dependence and overall normalisation, whereas the dependence on Q^2, M^2, p_t^2 and the quark mass comes out very similar.

Coming back to the calculation in the LN model one finds that the contribution of diagram (a) in Fig. 2 plus the one with reversed quark charge flow is obtained by replacing both L_1 and L_2 with $9\beta_0^2/(4\pi)$, so that only the first term in the brackets of (14) survives. Except for the root of $\alpha_s(\lambda^2)/\alpha_s^{(0)}$ this is just the DL model expression (10), (11) with the form factor (2) set equal to 1. In the DL model this form factor ensures the decrease of the amplitude at large p_t^2, while in the LN model the same effect is achieved by the contribution of diagram (b) and its analogue with opposite quark charge flow. In the LN model one can of course also calculate with the polarisations ε in Feynman gauge instead of $\tilde{\varepsilon}$ since the sum of the four diagrams is gauge invariant. If one does this [4] then the diagrams of type (a) give contributions to the amplitude for longitudinal photons that do not vanish at $Q^2 \to 0$, which are exactly cancelled by the diagrams of type (b). In the DL model the global factor $[f(\hat{t} - m_q^2) + f(\hat{u} - m_q^2)]/2$ cannot achieve this and one is left with an unphysical behaviour at $Q^2 \to 0$ when working in Feynman gauge.

Comparing (10) with (14) we see that the expression in the DL model can be obtained from the LN results by replacing both loop integrals L_1 and L_2 by K and by dropping the ratio $\alpha_s(\lambda^2)/\alpha_s^{(0)}$ of strong couplings at different scales in the LN expression. This ratio gives a different normalisation as a function of the scale λ^2, whereas the differences between the loop integrals (15) and the combination of form factors (11) have more subtle effects as we shall now see.

At $t = 0$ the longitudinal polarisation vector reads $\tilde{\varepsilon}_0^\mu = Q/(\Delta \cdot q) \cdot \Delta^\mu$ to leading order in ξ^{-1}, and it is easy to see that the sum of the first and the second term in the brackets of (14) gives an amplitude proportional to L_2 for longitudinal photons. For transverse photons the amplitude is proportional to L_1 in the case $m_q = 0$ when only the first term contributes, and it involves both loop integrals for finite quark mass due to the third term. These differences are all absent in the DL result, where the effect of the form factor is the same for all photon polarisations.

To illustrate the differences let us compare the values of L_1, L_2 and K in some kinematical limits, keeping in mind that there is also an additional root of $\alpha_s(\lambda^2)/\alpha_s^{(0)}$ in (14). We always assume that $\mu_0^2 \ll Q^2 + M^2$ so that from (2), (11) one has

$$K = \frac{9\beta_0^2}{8\pi} \, \frac{\mu_0^2}{\mu_0^2 + \lambda^2} \qquad (18)$$

up to terms suppressed by $\mu_0^2/(Q^2 + M^2)$.

For $\lambda^2 \gg \mu_0^2$ a reasonable approximation of the loop integrals L_1, L_2 is obtained by expanding the weighting functions f_1, f_2 around $l^2 = 0$, and with (13) one finds

$$\begin{aligned} L_1 &\approx \frac{9\beta_0^2\mu_0^2}{8\pi\lambda^2} \cdot 2 \left(1 - \frac{p_t^2}{\lambda^2}\right) \quad , \\ L_2 &\approx \frac{9\beta_0^2\mu_0^2}{8\pi\lambda^2} \cdot \left(1 - \frac{2p_t^2}{\lambda^2}\right) \quad , \\ K &\approx \frac{9\beta_0^2\mu_0^2}{8\pi\lambda^2} \quad . \end{aligned} \qquad (19)$$

In the case $p_t^2 \gg m_q^2$, which is relevant for jet production, one has $\lambda^2 \approx p_t^2/(1-\beta)$ so that due to the particular choice of form factor in (2) the p_t^2-dependence of the amplitude is the *same* in the two models for all photon polarisations. The β-dependence is however different: in the LN model one finds a zero of the longitudinal amplitude around $\beta = 1/2$ and a suppression of the transverse one at small β, both phenomenologically important effects [10,4] which are not found in the DL model.

In the opposite limit, $p_t^2 \ll m_q^2$ at $\lambda^2 \gg \mu_0^2$, which is relevant for heavy flavour production, one has $p_t^2/\lambda^2 \ll 1$ and thus $L_2 \approx K$. In the various $\gamma^* p$ cross sections ((27) of [10]) one finds that terms going with L_1 are suppressed by powers of p_t/m_q compared with those only depending on L_2, so that in this limit the two models lead to the same results, apart from the ratio of strong coupling constants.

When both p_t and m_q are small so that $\lambda^2 \ll \mu_0^2$ the weighting functions f_1 and f_2 tend to 1 and we have

$$L_1, L_2 \approx \frac{9\beta_0^2}{4\pi} \quad , \qquad K \approx \frac{9\beta_0^2}{8\pi} \quad , \qquad (20)$$

so that in this limit the DL amplitude is just half of that in the LN model, given that $\alpha_s(\lambda^2)/\alpha_s^{(0)}$ in the LN expression should be replaced with 1 at small λ^2.

2.3 The longitudinal diffractive structure function in the DL model

As an application of our modification of the DL model we calculate the integrated cross section for diffractive $q\bar{q}$-production, which can be taken as an approximation for the inclusive diffractive cross section – except in the region of small β where diffractive states with additional gluons are known to be important. Expressing our result in terms of the diffractive structure functions $F_T^{D(4)}$ and $F_L^{D(4)}$, conventionally defined by

$$\begin{aligned} \frac{d\sigma_{ep}}{dx\, dQ^2\, d\xi\, dt} &= \frac{4\pi\alpha_{em}^2}{xQ^4} \Big[\left(1 - y + y^2/2\right) F_T^{D(4)} \\ &\qquad + (1-y)\, F_L^{D(4)} \Big] \quad , \\ F_2^{D(4)} &= F_T^{D(4)} + F_L^{D(4)} \quad , \end{aligned} \qquad (21)$$

we obtain for $Q^2 + M^2 \gg \mu_0^2$

$$F_{T,L}^{D(4)}(\xi, \beta, Q^2, t) = f_{I\!P}(\xi, t) \cdot F_{T,L}^{I\!P}(\beta, Q^2, t) \qquad (22)$$

with

$$f_{I\!P}(\xi,t) = \frac{9\beta_0^2}{4\pi^2}\,\xi^{1-2\alpha_{I\!P}(t)}\left[F_1(t)^2 - \frac{t}{4m_p^2}F_2(t)^2\right] \tag{23}$$

and

$$F_T^{I\!P}(\beta,Q^2) = \frac{4}{3}\cdot\frac{3\beta_0^2\mu_0^2}{8\pi^2}\left\{\beta(1-\beta) + O\left(\frac{\beta\mu_0^2}{Q^2}\right)\right\} \quad ,$$

$$F_L^{I\!P}(\beta,Q^2) = \frac{4}{3}\cdot\frac{3\beta_0^2\mu_0^2}{8\pi^2}\left\{4\beta^3\,\frac{\mu_0^2}{Q^2}\left[\ln\left(\frac{Q^2}{\beta\mu_0^2}\right) - 1 + \frac{\beta\mu_0^2}{Q^2}\right]\right.$$
$$\left. + O\left(\left(\frac{\beta\mu_0^2}{Q^2}\right)^3\right)\right\} \quad , \tag{24}$$

where we have summed over massless quarks $u, \bar{u}, d, \bar{d}, s, \bar{s}$. We have made the approximation $t = 0$ in $F_T^{I\!P}$ and $F_L^{I\!P}$ but kept the strong t-dependence from the pomeron trajectory and the elastic form factors in $f_{I\!P}$. The terms of higher order in (24) give only small corrections: we find that the effect of the terms of order $\beta\mu_0^2/Q^2$ in $F_T^{I\!P}$ is below 3% for $Q^2/\beta \geq 5$ GeV^2 and that the approximation for $F_L^{I\!P}$ is better than 4% for $Q^2/\beta \geq 10$ GeV^2.

The result for $F_T^{I\!P}$ is what has long been obtained by DL [1]. It shows a scaling behaviour while the longitudinal structure function is power suppressed in $1/Q^2$, with $Q^2 \cdot F_L^{I\!P}$ having only a weak logarithmic dependence on Q^2. As $\beta \to 1$ the transverse structure function vanishes like $1-\beta$ whereas the longitudinal one goes to a constant. All these features are also found in the LN model [11]. In the DL model the ratio $F_L^{I\!P}/F_T^{I\!P}$ turns out to be numerically rather big even at moderately large β: for Q^2 between 10 GeV^2 and 30 GeV^2 it becomes equal to 1 at β between 0.65 and 0.8. Taking $Q^2 = 25$ GeV^2 one finds $F_2^{I\!P}$ rising all the way up to $\beta = 1$, with the increase of $F_L^{I\!P}$ overcompensating the decrease of $F_T^{I\!P}$ for $\beta > 1/2$. This looks difficult to reconcile with what is seen in the HERA data [13].

In the LN model one also finds that $F_L^{I\!P}$ becomes numerically important at large β where $F_T^{I\!P}$ goes to zero [11], but there the effect is much less dramatic. This is because of the extra factor of the running strong coupling at the scale $\lambda^2 = p_t^2/(1-\beta)$. It leads to a suppression at large β which is stronger for $F_L^{I\!P}$ than for $F_T^{I\!P}$ since the former is less dominated by small p_t^2 than the latter. It thus seems that the scale dependence of the strong coupling which appears in the two-gluon calculation leads to a more realistic prediction for the longitudinal structure function.

3 Exclusive production of vector mesons

The first process for which the DL and LN models have been compared and where their close relation was observed [3] was elastic electroproduction of a vector meson $V = \rho, \phi, J/\psi, \ldots$,

$$\gamma^*(q) + p(p) \to V(q') + p(p') \quad . \tag{25}$$

It turns out that the DL model in its original form also violates electromagnetic current conservation in this process, as was already remarked in [14]. For the $\gamma^* p$ cross section one obtains

$$\frac{d\sigma}{dt} = \frac{1}{16\pi W^4}\sum_{spins}{}' \,|\mathcal{M}|^2 \tag{26}$$

with

$$\mathcal{M} = \frac{48M^2}{e}\sqrt{\frac{3\pi\Gamma_{e^+e^-}}{M}}\,\frac{j\cdot q}{2p\cdot q}\,\xi^{-\alpha_{I\!P}(t)}\,\mathcal{P}_{DL}(\varepsilon,\varepsilon')$$
$$\times\frac{\beta_0^2\mu_0^2}{Q^2+M^2-t+2\mu_0^2} \quad , \tag{27}$$

where M denotes the meson mass, $\Gamma_{e^+e^-}$ its decay width into e^+e^-, and ε and ε' the respective polarisation vectors of photon and meson. The polarisation dependence is given by

$$\mathcal{P}_{DL}(\varepsilon,\varepsilon') = \frac{(p\cdot\varepsilon)\,(\Delta\cdot\varepsilon') - (\Delta\cdot\varepsilon)\,(p\cdot\varepsilon') + (\varepsilon\cdot\varepsilon')\,(p\cdot q)}{p\cdot q} \quad , \tag{28}$$

where we have used $\varepsilon'\cdot q' = 0$ but not made any assumption about ε so that this expression holds in any photon gauge. As announced $\mathcal{P}_{DL}$ does not vanish for the unphysical polarisation $\varepsilon = \varepsilon_3$. Working in the collision c.m. and defining polarisation vectors with respect to the $p-q$ axis for the photon and with respect to the $p'-q'$ axis for the meson we find in particular that for the transition of a longitudinal photon to a longitudinal vector meson one has

$$\mathcal{P}_{DL}(L,L) = \frac{Q^2 - M^2 + t}{2MQ} \tag{29}$$

in Feynman gauge, with an unphysical behaviour at small Q. In the LN model one finds[2]

$$\mathcal{M} = \frac{48M^2}{e}\sqrt{\frac{3\pi\Gamma_{e^+e^-}}{M}}\,\frac{j\cdot q}{2p\cdot q}\xi^{-\alpha_{I\!P}(t)}\sqrt{\frac{\alpha_s(\lambda^2)}{\alpha_s^{(0)}}}\mathcal{P}_{LN}(\varepsilon,\varepsilon')$$
$$\times\frac{8}{9}\int d^2l_t[\alpha_s^{(0)}]^2 D\left((\mathbf{l}_t - \mathbf{\Delta}_t/2)^2\right) D\left((\mathbf{l}_t + \mathbf{\Delta}_t/2)^2\right)$$
$$\times\frac{(\mathbf{l}_t^2 - \mathbf{\Delta}_t^2/4)}{Q^2+M^2-t+4(\mathbf{l}_t^2 - \mathbf{\Delta}_t^2/4)} \quad , \tag{30}$$

where Δ_t is the transverse part of Δ with respect to p and q, and

$$\mathcal{P}_{LN}(\varepsilon,\varepsilon') = \tag{31}$$
$$\frac{(p\cdot\varepsilon)\,(\Delta\cdot\varepsilon') - (\Delta\cdot\varepsilon)\,(p\cdot\varepsilon') + (\varepsilon\cdot\varepsilon')\,(p\cdot q) + \xi\,(p\cdot\varepsilon)\,(p\cdot\varepsilon')}{p\cdot q} \, .$$

Again this expression is valid in any photon gauge. If we require $Q^2 + M^2 \gg \mu_0^2$ and remember that the typical gluon virtualities in the integrals (13) are of order μ_0^2

[2] Our expression for $\mathcal{M}$ differs from those in [3] and [9,15] by numerical factors. Making the replacement (17) we agree with the result of [16]

then the terms after $\mathcal{P}_{DL}$ and $\mathcal{P}_{LN}$ in (27) and (30) are equal at $t = 0$; their difference will only become important when $-t$ is of order μ_0^2. Apart from this and from the root of $\alpha_s(\lambda^2)/\alpha_s^{(0)}$, where in analogy to (16) one would now choose $\lambda^2 = (Q^2 + M^2)/4$, the amplitudes in the two models then differ only by their polarisation factors $\mathcal{P}_{DL}$ and $\mathcal{P}_{LN}$. The latter has an extra term $\xi\,(p \cdot \varepsilon)\,(p \cdot \varepsilon')$ in the numerator, which was overlooked in [3,9] and recently reported in [15]. It precisely restores gauge invariance and guarantees a reasonable small-Q^2 behaviour

$$\mathcal{P}_{LN}(L,L) = \frac{Q}{M} \tag{32}$$

for the transition from a longitudinal photon to a longitudinal meson. (29) and (32) do not even agree in the limit $Q^2 \gg M^2$, where $\mathcal{P}_{LN}(L,L)$ is twice as large as $\mathcal{P}_{DL}(L,L)$. For transverse photons on the other hand $\mathcal{P}_{DL}$ gives the same as $\mathcal{P}_{LN}$. Note however that, unlike in the case of the diffractive structure function discussed in Sect. 2.3, transverse photon polarisation in vector meson production is suppressed at large Q^2; with (32) one finds $\sigma_L/\sigma_T = Q^2/M^2$ for the ratio of longitudinal and transverse cross sections at all Q^2 and t.

If we work in the gauge $A \cdot p = 0$ then the extra term in $\mathcal{P}_{LN}$ vanishes identically so that $\mathcal{P}_{DL}$ agrees with $\mathcal{P}_{LN}$ for all physical photon polarisations. Defining the DL model in this gauge we thus find that for $Q^2 + M^2 \gg \mu_0^2$ and $-t \ll \mu_0^2$ it gives the same result as the LN approach up to the scale dependent ratio $\alpha_s(\lambda^2)/\alpha_s^{(0)}$. This corresponds to what we found for $q\bar{q}$-production in the limit $m_q^2 \gg p_t^2$ and $\lambda^2 \gg \mu_0^2$ in Sect. 2.2; note that following [1] in the calculation of meson production we have used a constituent mass of $M/2$ for the quarks and neglected their transverse momentum in the meson so that kinematically the two processes are equivalent.

Let us finally compare the two gauges $A \cdot n = 0$ with $n = p$ and $n = \Delta$ at finite t. With Δ as gauge fixing vector one does *not* reproduce the results of the LN calculation since the extra term in $\mathcal{P}_{LN}$ does not always vanish. This happens when the photon is transversely polarised in the scattering plane while the meson is longitudinal. $\mathcal{P}_{LN}$ is then negligible in the small-ξ limit whereas $\mathcal{P}_{DL} = -\sqrt{|\,t\,|}/M$ is not: the DL model defined in this gauge thus violates s-channel helicity conservation. Both from this point of view and in order to reproduce as closely as possible the two-gluon result the choice $n = p$ therefore seems preferable to us.

4 Discussion

4.1 Contact terms

The point of view underlying our detailed comparison of the DL and the LN models is that diffraction can be described in QCD by multi-gluon exchange in the colour singlet channel, and that the LN model is a simple implementation of this idea. The DL model, on the other hand, looks more like an effective theory in which gluon degrees of freedom are no longer explicitly present.

It can be shown that in the high-energy limit the coupling of two t-channel gluons to a single quark line can be written in terms of the quark vector current [2]. Thus the diagram in Fig. 2 (a) is equivalent to the one in Fig. 1 (a) calculated without a form factor for the quark-pomeron coupling, as we reported in the sequel of (17). Technically this is seen as follows: for the upper quark line in Fig. 2 (a) one has $\not{p}\,(\not{k}+m_q)\,\not{p} \approx 2(p\cdot k)\,\not{p}$ to leading order in energy, where k denotes the momentum of the quark line between the two quark-gluon vertices, while p is the dominant part of the gluon polarisation at each vertex and becomes the dominant "pomeron polarisation" in the DL model. One can also explicitly see how this gives the correct $C = +1$ quantum number of the exchange: reversing the charge flow of the quark line but keeping the flow of its momentum unchanged one obtains $\not{p}\,(-\not{k} + m_q)\,\not{p} \approx -2(p \cdot k)\,\not{p}$; the relative minus sign is precisely what is put in by hand between the two diagrams in the DL model.

The diagram in Fig. 2 (b) has a different topology than the diagrams of Fig. 1, and its contribution to the amplitude cannot be entirely rewritten in terms of an effective quark-pomeron coupling with a form factor. It is plausible to assume that a part of its contribution will have the structure of a contact interaction between the quark line, the pomeron and the photon; note that the generation of contact interactions is well known in effective field theories. Such a contact interaction can have a rather rich structure and we are not attempting here to construct a corresponding extension of the DL model, in which gauge invariance is restored. Let us however make the hypothesis that such an extension can be found, where the sum of all diagrams for a process conserves the electromagnetic current. Terms with an unphysical $1/Q$-behaviour from the diagrams of Fig. 1 will then be cancelled by terms from the diagram with a quark-pomeron-photon contact interaction.

Now the procedure of gauge fixing we proposed in Sect. 2.1 can be seen in a different light. In a gauge invariant theory one can of course work in *any* photon gauge; some terms in the amplitude will be "shifted" between different diagrams when the gauge is changed. There will be gauges for which the diagrams with and without contact term are separately well behaved in Q; an example is just the gauge $A \cdot n = 0$ with a suitably chosen vector n. Leaving out the contribution from the contact term (because we do not know its form) we then make an error, but this error is finite in the photoproduction limit. The error will be different for different choices of n; and whether there is a particularly good choice or even one where the contact term is completely eliminated we can of course not determine without knowing the specific form of the contact interaction. Nevertheless there are certain criteria: our discussion at the end of Sect. 3 shows for instance that there are "unfortunate" choices for which s-channel helicity in meson production is not conserved by the diagrams without contact terms; in a model reproducing the two-gluon exchange results such helicity violating terms will then only be cancelled when contact interaction diagrams are taken into account.

4.2 Comparison with other approaches

Several approaches to the problem of coupling the pomeron to quarks have been made in the literature. In [17] and later in [18] the pomeron was treated exactly like a photon, with a γ^μ-coupling but without a form factor and without any relative minus signs between diagrams. By construction this model does not have any trouble with gauge invariance, but at the price of actually describing $C = -1$ instead of $C = +1$ exchange. It can for instance not describe the forward $\gamma^* p$ scattering amplitude, i.e. the inclusive proton structure function at small x, since the coupling of three vector particles (two γ^* and the pomeron) to a quark loop vanishes due to Furry's theorem. For diffractive $q\bar{q}$-production it gives a p_t-behaviour that is remarkably different from the one in the DL calculation. The DL model with the form factor (2), as well as the LN model, gives a transverse $\gamma^* p$ cross section $d\sigma_T/(dp_t^2\, dM^2)$ that falls off approximately like $1/p_t^4$ at large p_t and is finite as p_t goes to zero. In the approach of [17,18] the decrease at large p_t is much slower, and for massless quarks one has a $1/p_t^2$-singularity at zero p_t. This has important consequences for the description of the leading twist part of F_2^D in terms of diffractive parton densities, whose evolution equation then has an *inhomogeneous* term – just as in the case of parton densities in the photon – which is intimately related with this collinear singularity in the transverse $q\bar{q}$ cross section. Such an inhomogeneous term will not be generated in the DL model. We also remark that unlike the DL and LN approaches the pomeron model of [17,18] gives a leading twist contribution to the longitudinal structure function F_L^D, going like $\beta^2(1-\beta)$, in strict analogy with the photon structure function [19].

The model of [20] assumes a pointlike quark-pomeron coupling through the scalar instead of the vector current of the quarks, i.e. it treats the pomeron as a scalar exchange. This is again gauge invariant, and the scalar current has the correct quantum numbers to model the pomeron, in particular it is $C = +1$. Taking the produced quarks as massless one obtains a transverse $q\bar{q}$ cross section going like $1/p_t^2$ at large and at small p_t – the divergence will again lead to an inhomogeneous evolution equation for diffractive quark densities – and a longitudinal $q\bar{q}$ cross section that vanishes at $t = 0$. We note that for massless quarks a scalar quark-pomeron coupling flips the quark helicity, in contrast to models with a vector coupling which conserves it. The helicity of massless quarks is of course also conserved by multi-gluon exchange, at least if one considers the γ^μ-structure of the perturbative quark-gluon vertex to be relevant in this context.

The starting point of [14] is a pomeron with a vector coupling and a sign factor to ensure $C = +1$ exchange; a scalar term accompanied with specific sign instructions is then added to the γ^μ-part of the coupling in order to restore current conservation in vector meson production. For a programme of investigating the general structure of the pomeron-quark vertex we refer to [21].

An effective theory for high energy quark-quark scattering, starting from multi-gluon exchange, has been formulated in [22]. The current for $C = +1$ exchange has the structure $\bar{\psi}\gamma^\mu \overset{\leftrightarrow}{D}_{\nu_1} \overset{\leftrightarrow}{D}_{\nu_2} \ldots \overset{\leftrightarrow}{D}_{\nu_{2n+1}} \psi$, where the odd number of covariant derivatives gives the correct quantum numbers of the pomeron. The Dirac matrix γ^μ leads to the same quark spin structure of the coupling as in the DL model, while the derivatives guarantee the correct signs between diagrams when the quark charge flow is reversed, in a manner similar to the explicit example of two-gluon exchange discussed in the previous subsection.

5 Summary

There are several possibilities to describe the pomeron coupling to quarks in a phenomenological model. The Donnachie-Landshoff model chooses a γ^μ-coupling and implements the $C = +1$ parity of the pomeron by introducing appropriate relative minus signs between diagrams. In order to obtain phenomenologically reasonable results the pomeron-quark coupling *cannot* be pointlike in this model. We point out that multi-gluon exchange in QCD motivates a form factor behaviour of the quark-pomeron coupling[3] as well as helicity conservation for massless quarks, which is ensured by the γ^μ-vertex in the DL model.

The price to be paid for introducing minus signs "by hand" in this model is that the electromagnetic current is not conserved. We propose to define the model by specifying a noncovariant gauge $A \cdot n = 0$ in which a good Q^2-behaviour is guaranteed for all physical photon polarisations in the limit where the photon becomes real. The choice of a gauge fixing vector n is not unique and has to be motivated, for instance by comparing predictions with more elaborate models such as two-gluon exchange. Our preferred choice here is $n = p$.

A way to restore current conservation is the introduction of contact terms between quark, photon and pomeron lines, which seems natural if one thinks of the model as an "effective theory" of multi-gluon exchange. To construct such terms is beyond the scope of this work, but we note that in such a model one can make a suitable choice of photon gauge to minimise the contribution of contact term graphs, so that reasonable results may be obtained even when they are left out. Our choice of gauge then assures in particular a good Q^2-behaviour of the diagrams with and without contact terms separately.

We have applied the DL model in the gauge $A \cdot p = 0$ to diffractive $q\bar{q}$-production at $t = 0$ and compared in some detail its results with those of the LN model of two-gluon exchange. While their predictions differ in detail there is

[3] This should at least hold for the part of the coupling that gives the leading twist contribution to the diffractive structure function F_2^D. In [23] a possible pointlike structure of the pomeron in perturbative QCD was discussed at the level of *nonleading* twist

a strong similarity between the two models, in particular for the p_t-dependence, which leads to the prediction that F_T^D is of leading twist whereas F_L^D is not. For vector meson production we find almost identical results in the two models when Q^2 or the meson mass is large and t is small, provided one takes p as gauge fixing vector. Taking Δ instead would lead to a violation of s-channel helicity conservation, at variance with two-gluon exchange.

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